

Form factors and charge radii in a QCD inspired potential model using the Variationally Improved Perturbation Theory (VIPT)

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Abstract

We use VIPT in calculating the elastic form factors and charge radii of D, D_s, B, B_s and B_c mesons in a QCD inspired potential model. For that we use linear cum Coulombic potential and opt firstly the Coulombic part as parent and then linear part as parent. The results show that charge radii and form factors are quite small for Coulombic parent case than the linear parent. Also, the analysis leads to a lower as well as upper bound on the four momentum transfer Q^2 hinting at a workable range of Q^2 within this approach which may be useful in future for experimental analysis. Comparison of both the options prefer the linear parent.

Keywords: VIPT, form factor, charge radii .

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1 Introduction

The potential model description in the nonperturbative regime of QCD is tremendously successful in providing both qualitative and quantitative description of the hadron spectrum and the decay modes. On this basis, we have pursued a Non Relativistic Constituent Quark Model (NRQM) [1-4] for mesons containing a heavy (light) quark and a light (heavy) antiquark. Although non relativistic in nature, relativistic effect is to be introduced from outside [5,6] due to the light quarks involved. Basically, the model relies on the work of Rujula, Georgi and Glashow [7] who used a non-relativistic treatment of potential model which was quite successful in the description of different hadronic properties. We have solved the Schrödinger equation for the spin independent Fermi-Breit Hamiltonian (used in the work of Rujula et al [7]) consisting of the linear cum Coulombic potential for the ground state [2]. The solution i.e. wavefunction has been obtained using different approximation methods like Dalgarno method [8] and Variationally Improved Perturbation Theory (VIPT) [9-11] which is then used in predicting the Isgur-Wise (I-W) function [2,3,4,12,13], mass, decay constant, charge radii [1] etc. We note that with the linear cum Coulomb potential of QCD we have two options in choosing parent or child (i.e. perturbation) - (i) first, we can consider the linear one as the perturbation (i.e. the Coulombic one as the parent) and then (ii) Coulombic one as the perturbation (i.e. linear one as the parent). As we have already successfully used VIPT for both the options in the calculation of slope and curvature of I-W function for D, D_s, B, B_s and B_c mesons [12,13], so extending it for the prediction of elastic form factors and charge radii (which provide important insight on the distribution of different charge constituents of a hadron) definitely makes a sense.

It is well known that the form factor and charge radius are dependent on the momentum transform of the wavefunction. So, getting an appropriate wavefunction is very essential for a fruitful analysis. With the success of VIPT in the calculation of Isgur-Wise function as pointed above [12,13], one can expect a similar success here also. It is worthwhile to note that while investigating the form factor, one must take into account of a proper range of four momentum transfer Q^2 . The Q^2 range usually determines the applicability of perturbative QCD (pQCD) or nonperturbative (npQCD). So, an accurate selection of Q^2 range within the nonperturbative approach is necessary which will also fall within the experimental regime. This facilitates a direct comparison between theory and experiment. This has been done both theoretically and experimentally since long [14-19] for the light π , K etc mesons. However, for the mesons which contain at least one heavy quark, very little have been investigated theoretically [20-22]. In the absence of any experimental data for them, our results may be helpful in future in the experimental set up regarding the Q^2 range.

As far as our model is concerned, the perturbative or nonperturbative regime of QCD can be interpreted [1] through the relativistic factor ' $\epsilon = 1 - \sqrt{1 - \frac{4\alpha_s}{3}}$ '. The reality constraint on the form factor $F(Q^2)$ leads to the condition $0 < \epsilon < 1$, where the case $\epsilon \rightarrow 0$ ($\epsilon \rightarrow 1$) corresponds to the perturbative (nonperturbative) limit of QCD. The $\epsilon \rightarrow 1$ limit demands large α_s or low Q^2 . So, discussing the nonperturbative effects of QCD with large confinement parameter b , we must consider the low Q^2 limit of α_s in this model. However, we have observed in Ref[1] that large value of $b (= 0.183 \text{ GeV}^2)$ prohibits the use of low Q^2 compelling one to involve with small α_s which corresponds to the perturbative regime and thus can't be accounted in this nonperturbative approach.

We reanalyze all these observations in this approach of VIPT for both the cases -linear or confinement part as perturbation and Coulombic part as perturbation. We will explore the possibility of incorporating significant value of α_s even with large confinement ($b = 0.183 \text{ GeV}^2$). This work will also check the status of both confinement and Coulombic part as perturbation and observed the consequences regarding the usable range of Q^2 to work with in the absence of experimental data for the said mesons. The calculations are done with a fixed value of α_s from V-scheme [23-25] with large confinement effect $b = 0.183 \text{ GeV}^2$ instead of variation in both. Even with this single value of α_s and b one can draw similar conclusion regarding the effective range of Q^2 . The calculated form factors are plotted graphically to show their variation with Q^2 for both the cases.

Basically, this work explores the possibility of improving the results for form factors and charge radii over those of Ref[1,22] with the help of VIPT. In the process, we also try to find a useful range of Q^2 which may be workable for the experimental investigations in the later course of time. Comparison of both the options is being made to arrive at a conclusion in using VIPT.

The rest of the paper is organized as follows : section 2 contains the formalism, section 3 the result and calculation while section 4 includes the discussion and conclusion.

2 Formalism

2.1 VIPT with Coulombic potential as parent

2.1.1 Wavefunction

We briefly reformulate the VIPT with the expression for the wavefunction [10, 12] corrected upto the first order of j^{th} state given by :

$$\psi_j = \psi_j^{(0)} + \sum_{k \neq j} \frac{\int \psi_k^{(0)*} H'_{P,j} \psi_j^{(0)} dv}{E_j^{(0)} - E_k^{(0)}} \quad (1)$$

where P' is the variational parameter (which is later optimized w.r.t. energy) considered instead of physical parameter P . For the Coulombic part as parent, the physical parameter is $\alpha = \frac{4\alpha_s}{3}$ and the optimized variational parameter is $\bar{\alpha}'$ [12]. As done in Ref[12], we consider the wave function for triple term consideration of the summation given by equation (1) above and rewrite the equation (viz. eq(45) of Ref[12]) with the subscript 10 ($n = 1, l = 0$) being replaced by T :

$$\psi_T = \psi_{10}^{(0)} - A \left(1 - \frac{\mu \bar{\alpha}'_{10} r}{2} \right) e^{-\frac{\mu \bar{\alpha}'_{10} r}{2}} + B \left(1 - \frac{2\mu \bar{\alpha}'_{10} r}{3} + \frac{2\mu^2 \bar{\alpha}'_{10}{}^2 r^2}{27} \right) e^{-\frac{\mu \bar{\alpha}'_{10} r}{3}} + D' \left(\frac{1}{4} - \frac{3\mu \bar{\alpha}'_{10} r}{16} + \frac{\mu^2 \bar{\alpha}'_{10}{}^2 r^2}{32} - \frac{\mu^3 \bar{\alpha}'_{10}{}^3 r^3}{8 \times 96} \right) e^{-\frac{\mu \bar{\alpha}'_{10} r}{4}} \quad (2)$$

where the different parameters are given by:

$$c'_1 = \frac{\mu \bar{\alpha}'_{10}}{\pi^{\frac{1}{3}}} \quad (3)$$

$$A = \frac{4\sqrt{\mu}}{3\sqrt{\pi} (\bar{\alpha}'_{10})^{\frac{1}{2}}} \left[\frac{4\mu \bar{\alpha}'_{10} (\alpha - \bar{\alpha}'_{10})}{27} - \frac{32b}{81\mu \bar{\alpha}'_{10}} \right] \quad (4)$$

$$B = \frac{\sqrt{\mu}}{\sqrt{\pi} (\bar{\alpha}'_{10})^{\frac{1}{2}}} \left[\frac{3\mu \bar{\alpha}'_{10} (\alpha - \bar{\alpha}'_{10})}{64} - \frac{27b}{256\mu \bar{\alpha}'_{10}} \right] \quad (5)$$

and

$$D' = \frac{(\mu \bar{\alpha}'_{10})^{\frac{3}{2}}}{\sqrt{\pi}} \left[\frac{36 (\alpha - \bar{\alpha}'_{10})}{15625 \bar{\alpha}'_{10}} - \frac{384b}{78125 \mu^2 \bar{\alpha}'_{10}{}^3} \right] \quad (6)$$

The wavefunctions for single and double term consideration can be obtained by putting $B = D' = 0$ and $D' = 0$ respectively in the same equation (2).

However we will consider the relativistic version ($\epsilon \neq 0$) of the above wave function viz. [12]

$$\psi_{T,Rel}(r) = \psi_T(r \mu \bar{\alpha}'_{10})^{-\epsilon} \quad (7)$$

The relativistic factor ϵ is given by [1]:

$$\epsilon = 1 - \sqrt{1 - \frac{4\alpha_s}{3}} \quad (8)$$

2.1.2 The elastic charge form factor and charge radii

The form factor can be expressed as [26]:

$$eF(Q^2) = \sum \frac{e_i}{Q_i} \int_0^{+\infty} r |\psi_{T,Rel}(r)|^2 \sin Q_i r dr \quad (9)$$

where

$$Q_i = \frac{\sum_{j \neq i} m_j Q}{\sum m_i} \quad (10)$$

Putting (2) and (7) in (9) we get the form factor as :

$$eF(Q^2) = \sum e_i N_3^2 \Gamma(3 - 2\epsilon) [q_1 + q_2 + q_3 + q_4 + q_5 + q_6 + q_7 + q_8 + q_9 + q_{10}] \quad (11)$$

where N_3^2 is the same normalization constant as appeared in equation(54) of Ref[12] and the different $q_i(Q_i)$ s ($i = 1, 2, \dots, 10$) are defined in the Appendix.

The charge radius is derived as [1] :

$$\langle r^2 \rangle = - \frac{d(eF(Q^2))}{dQ^2} \Big|_{Q^2=0} \quad (12)$$

$$= N_3'^2 \Gamma(3 - 2\epsilon) [r_1 + r_2 + r_3 + r_4 + r_5 + r_6 + r_7 + r_8 + r_9 + r_{10}] \quad (13)$$

where the different r_i s ($i = 1, 2, \dots, 10$) are defined in the Appendix.

2.1.3 Status of linear potential as perturbation

The momentum transform of equation (7) is [27, 28]:

$$\psi_{T,Rel}(Q^2) = \sum \frac{e_i}{Q_i} \sqrt{\frac{2}{\pi}} \int_0^{+\infty} r \psi_{T,Rel}(r) \sin Q_i r dr \quad (14)$$

$$= \sum e_i N_3' \sqrt{\frac{2}{\pi}} \Gamma(3 - 2\epsilon) [p_1 - p_2 + p_3 + p_4] \quad (15)$$

The p_i s which depend on Q_i^2, ϵ etc are given in the Appendix.

If linear potential is treated as perturbation then from equation(15) above the following inequality must be preserved:

$$p_1 > p_2 - p_3 - p_4 \quad (16)$$

This inequality leads to a lower limit of Q^2 namely Q_0^2 [1] above which one have to use the values of Q^2 . The Q_0^2 is determined from the condition:

$$p_1 = p_2 - p_3 - p_4 \quad (17)$$

Due to the quark mass dependence, Q_0^2 s have different values and they are shown in table 3. In the Dalgarno method approach [1], the lower limits Q_0^2 were large and the formalism failed to account for large confinement effect ($b = 0.183 GeV^2$) in the nonperturbative QCD regime where α_s values were taken to be large. Only in the limit $b \rightarrow 0$, the Q_0^2 values were lowered and the formalism worked for low Q^2 range [1]. In this method of VIPT, the values of Q_0^2 are shown to be quite small even with large confinement effect ($b = 0.183 GeV^2$) enabling us to work in the nonperturbative QCD regime with large α_s .

We also note that for single term consideration only p_2 exists on the RHS of inequality and for double term both p_2 and p_3 exist. We have also recorded the values of Q_0^2 for single and double term consideration in table 3.

2.2 VIPT with linear potential as parent

2.2.1 Wavefunction

As pointed in Ref[10,13] linear parent gives rise to Airy functions. The physical parameter is b and the optimized variational parameter is \bar{b} . We reproduce the analogous wavefunction in this case also for three term consideration of eq.(1) as was for the Coulombic parent:

$$\begin{aligned}
\psi_T = N''[\psi^{(0)} + & \frac{(2\mu)^{\frac{1}{3}}}{(\rho_{02} - \rho_{01})\bar{b}^{\frac{2}{3}}} \left((b - \bar{b}') < r >_{2,1} - \alpha < \frac{1}{r} >_{2,1} \right) \psi_{20}(r) + \\
& \frac{(2\mu)^{\frac{1}{3}}}{(\rho_{03} - \rho_{01})\bar{b}^{\frac{2}{3}}} \left((b - \bar{b}') < r >_{3,1} - \alpha < \frac{1}{r} >_{3,1} \right) \psi_{30}(r) \\
& + \frac{(2\mu)^{\frac{1}{3}}}{(\rho_{04} - \rho_{01})\bar{b}^{\frac{2}{3}}} \left((b - \bar{b}') < r >_{4,1} - \alpha < \frac{1}{r} >_{4,1} \right) \psi_{40}(r)] \quad (18)
\end{aligned}$$

where N'' is the normalization constant as appeared in equation(25) of Ref[13]. We note that for single (double) term consideration of equation(1) the third and fourth term (fourth term) is dropped from the equation (18) and normalization constants also changes to different one (Eq.17 and 21 of Ref [13]).

For this case also we take the relativistic version of the above wavefunction :

$$\psi_{T,Rel} = \psi_T (r\mu\bar{\alpha}'_{10})^{-\epsilon} \quad (19)$$

The zeros of the Airy function ρ_{0n} is given by eq.(11) of Ref[13] as:

$$\rho_{0n} = - \left[\frac{3\pi(4n-1)}{8} \right]^{\frac{2}{3}} \quad (20)$$

and

$$< r^k >_{n,n'} = N_n N_{n'} \int_0^{+\infty} r^k Ai \left((2\mu\bar{b}')^{\frac{1}{3}} r - \rho_{0n} \right) Ai \left((2\mu\bar{b}')^{\frac{1}{3}} r - \rho_{0n'} \right) dr \quad (21)$$

where $N_n, N_{n'}$ are the normalization constants for n and n' states respectively etc.

Like the expressions we have adopted the same values of b, \bar{b}', ρ_{0n} from Ref[13](see tables 1,2 there).

2.2.2 The elastic charge form factor and charge radii

Putting (18) and (19) in the definition of form factor (9) given above, the form factor is found to be :

$$eF(Q^2) = \sum e_i N''^2 \left[C - C' \frac{Q_i^2}{6} \right] \quad (22)$$

The coefficients C, C' s are given in table 4. They are of course different for single ,double or more than two term consideration. Numerical integrations are done in getting the above result.

The corresponding charge radius is obtained by using equation (18) and (19) in(12) which are recorded in table 4.

2.2.3 Status of Coulombic potential as perturbation

The momentum transform of (19) is:

$$\psi_{T,Rel}(Q^2) = \sum \frac{e_i}{Q_i} \sqrt{\frac{2}{\pi}} \int_0^{+\infty} r \psi_{T,Rel}(r) \sin Q_i r dr \quad (23)$$

$$\begin{aligned}
& = \sum e_i N'_3 \sqrt{\frac{2}{\pi}} \Gamma(3-2\epsilon) [p'_1 + p'_2 + p'_3 + p'_4 \\
& - \frac{Q_i^2}{6} (p'_5 + p'_6 + p'_7 + p'_8)] \quad (24)
\end{aligned}$$

The p'_i s ($i = 1, 2, \dots, 8$) are given in the Appendix.

If Coulombic potential is treated as perturbation then from equation(24) above the following inequality must be preserved:

$$p'_1 + p'_2 + p'_3 + p'_4 > \frac{Q_0^2}{6} (p'_5 + p'_6 + p'_7 + p'_8) \quad (25)$$

This inequality leads to a upper limit of Q^2 namely Q_0^2 below which one have to use the values of Q^2 .The Q_0^2 is determined from the condition:

$$p'_1 + p'_2 + p'_3 + p'_4 = \frac{Q_0^2}{6} (p'_5 + p'_6 + p'_7 + p'_8) \quad (26)$$

The different values of upper limit Q_0^2 s are shown in table 5.The corresponding values for single and double term consideration are also shown.

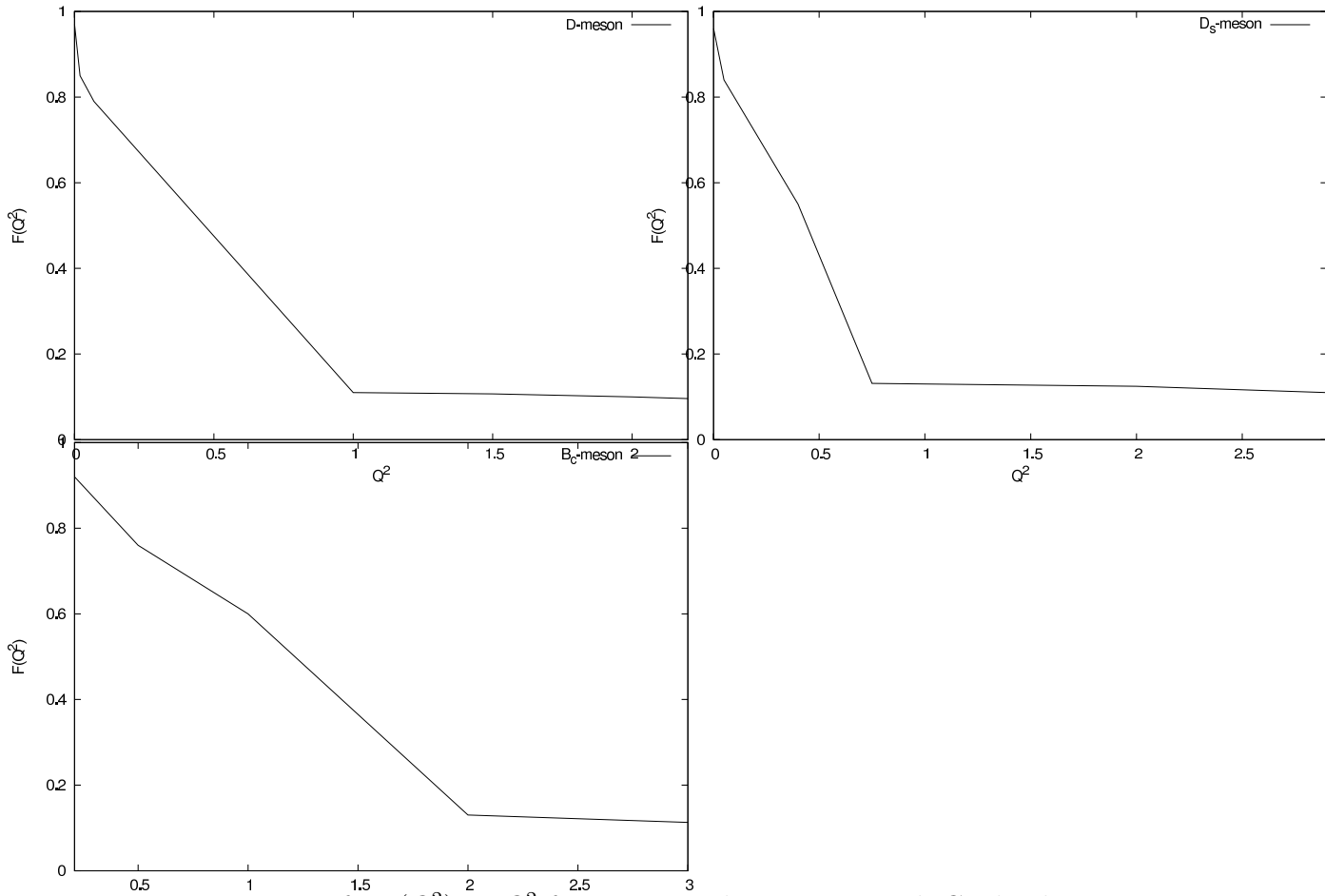


Figure 1: Variation of $eF(Q^2)$ vs Q^2 for D , D_s and B_c -meson with Coulombic parent .

3 Calculation and Results

We have listed \bar{b}' in table 1 while the lower and upper limit of Q_0^2 for single,double and triple term consideration are given in tables 3 and 5. In table 2 , we record the charge radii for single,double and triple term consideration for Coulombic potential as parent; whereas the same is recorded for linear potential as parent in table 4.The infinite mass consideration shown by the subscript ∞ is also included for triple (single) term consideration for Coulombic (linear) parent.The table

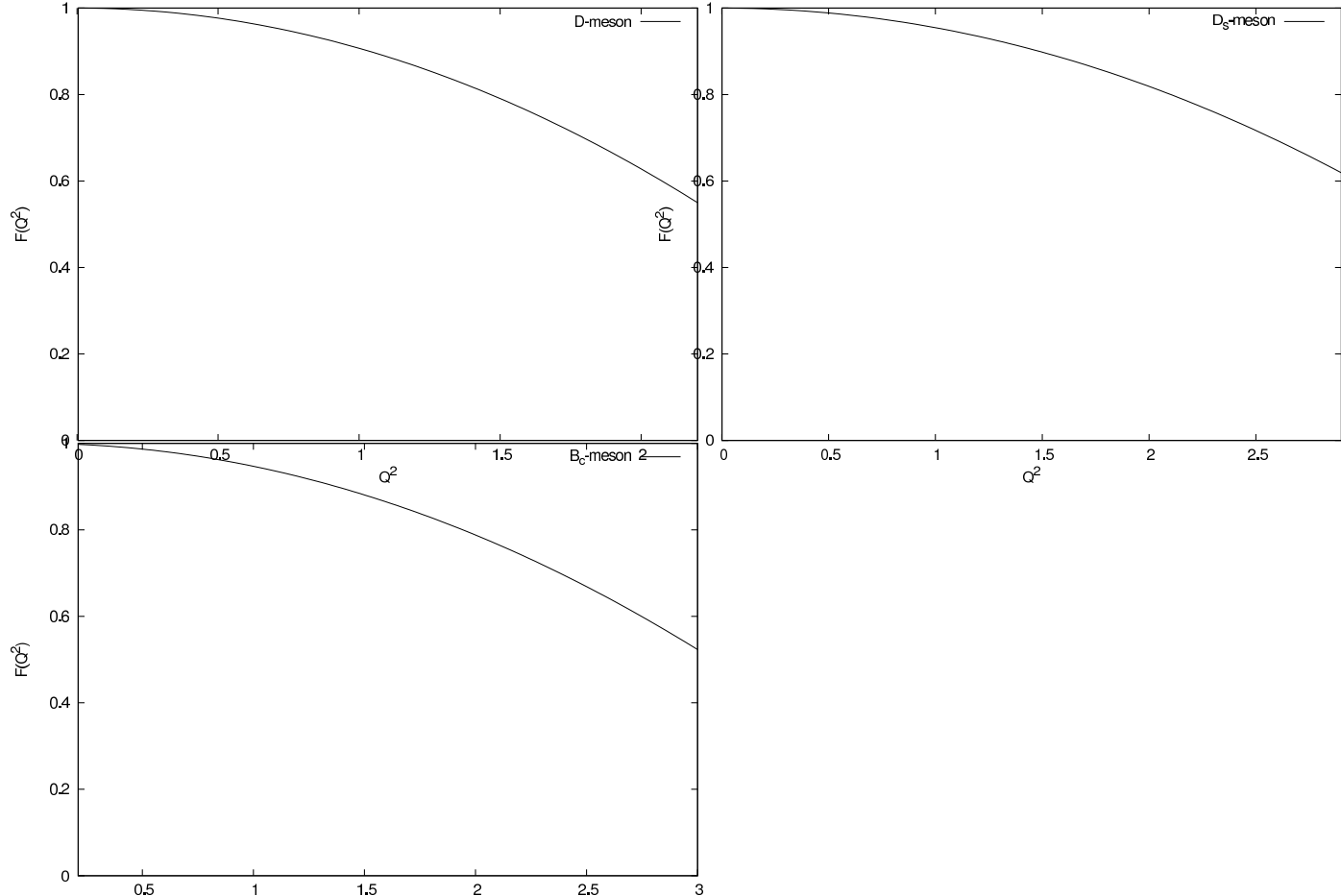


Figure 2: Variation of $eF(Q^2)$ vs Q^2 for D , D_s and B_c -meson with linear parent .

6 shows charge radii of different mesons obtained from other models and data. The α_s values are taken from the V -scheme [23, 24, 25] and the integrations are done numerically for all these calculations. The graphs show the variation $eF(Q^2)$ vs Q^2 for D , D_s and B_c mesons for both the options.

Table 1: Values of \bar{b}' with $b = 0.183GeV^2$ for relativistic case only. The values are the same as recorded in table 2 of Ref[13]

Mesons	Reduced mass μ	$\alpha = \frac{4\alpha_s}{3}$	\bar{b}' with relativistic effect
D	0.2761	0.924	16.24
D_s	0.368248	0.924	19.8
B	0.31464	0.348	5.587
B_s	0.4401	0.348	5.954
B_c	1.1803	0.348	8.103

Table 2: Values of charge radii for different mesons with Coulombic parent for single, double and triple terms in eq.(1). The subscripts ' S, D, T ' correspond to single, double and triple terms respectively whereas ' F ' means finite mass consideration. The infinite mass limit (subscript ∞ is used) is shown for the triple term alone .

Meson	D^0	D^+	D_s^+	B^+	B^0	B_s^0	B_c^+
$\langle r_{S,F}^2 \rangle^{\frac{1}{2}} \text{ in fm}$	-0.121	0.115	0.11	0.2545	-0.1822	-0.168	0.108
$\langle r_{D,F}^2 \rangle^{\frac{1}{2}} \text{ in fm}$	-0.119	0.112	0.106	0.2512	-0.1788	-0.164	0.105
$\langle r_{T,F}^2 \rangle^{\frac{1}{2}} \text{ in fm}$	-0.118	0.109	0.101	0.2464	-0.1736	-0.158	0.1034
$\langle r_{T,\infty}^2 \rangle^{\frac{1}{2}} \text{ in fm}$	-0.131	0.12	0.113	0.263	-0.186	-0.1742	0.1325

Table 3: Values of lower limit of four momentum transfer Q_0^2 with Coulombic parent taking single, double and triple terms in eq.(1). We have to use Q^2 values above these.

Meson	D^+	D^-	D_s^+	B^+	B^0	B_s^0	B_c^+
$Q_{0,S}^2$	0.0004	0.0004	0.001	0.053	0.053	0.075	0.211
$Q_{0,D}^2$	0.00036	0.00036	0.0009	0.052	0.052	0.072	0.209
$Q_{0,T}^2$	0.0003	0.0003	0.0007	0.05	0.05	0.07	0.205

Table 4: Values of charge radii for different mesons with linear parent for single,double and triple terms in eq.(1) . The subscripts ‘ S, D, T ’ correspond to single,double and triple terms respectively whereas ‘ F ’ means finite mass consideration .The infinite mass limit (subscript ∞ is used) is shown for the single term alone.

Meson	D^0	D^+	D_s^+	B^+	B^0	B_s^0	B_c^+
C_S	8.24	8.24	5.916	14.22	14.22	10.91	4.50
C'_S	2.266	2.266	1.173	7.71	7.71	4.52	0.78
$< r_{S,F}^2 >^{\frac{1}{2}} \text{ in fm}$	-0.197	0.1494	0.104	0.425	-0.2996	-0.2227	0.1125
C_D	13.1	13.1	8.1	26.7	26.7	16.5	5.2
C'_D	2.69	2.69	1.67	9.89	9.89	5.7	1.1
$< r_{D,F}^2 >^{\frac{1}{2}} \text{ in fm}$	-0.21	0.161	0.121	0.473	-0.336	-0.2489	0.127
C_T	18.14	18.14	11.18	43.09	43.09	25.56	6.636
C'_T	3.365	3.365	2.199	13.14	13.14	7.655	1.363
$< r_{T,F}^2 >^{\frac{1}{2}} \text{ in fm}$	-0.24	0.182	0.143	0.555	-0.391	-0.289	0.148
$< r_{S,\infty}^2 >^{\frac{1}{2}} \text{ in fm}$	-0.246	0.174	0.125	0.453	-0.32	-0.246	0.144

Table 5: Values of upper limit of four momentum transfer Q_0^2 with linear parent taking single,double and triple terms in eq.(1).We have to use Q^2 values lower than these.

Meson	D^+	D^-	D_s^+	B^+	B^0	B_s^0	B_c^+
$Q_{0,S}^2$	2.297	2.297	2.92	1.43	1.43	1.676	3.11
$Q_{0,D}^2$	2.1	2.1	2.67	1.31	1.31	1.56	2.89
$Q_{0,T}^2$	1.88	1.88	2.387	1.177	1.177	1.386	2.55

Table 6: Prediction of $\langle r^2 \rangle^{\frac{1}{2}}$ in fm for finite and infinite mass consideration in other models. The subscript 'F' (' ∞ ') means finite (infinite) mass limit.

Meson	D^0	D^+	D_s^+	B^+/B^-	B^0	$B_s^0/\overline{B_s^0}$	B_c^+/B_c^-
$\langle r_F^2 \rangle^{\frac{1}{2}}[9]$...	0.506	0.491	0.258(B^-)	...	0.256($\overline{B_s^0}$)	0.236(B_c^-)
$\langle r_F^2 \rangle^{\frac{1}{2}}[10]$	-0.551	0.43	0.352	0.612(B^+)	-0.432	-0.345(B_s^0)	0.207(B_c^+)
$\langle r_\infty^2 \rangle^{\frac{1}{2}}[10]$	-0.704	0.498	0.425	0.704(B^+)	-0.498	-0.425(B_s^0)	...(B_c^+)
$\langle r_F^2 \rangle^{\frac{1}{2}}[11]$	-0.484	0.366	0.355	1.72(B^+)	-1.21	-1.17(B_s^0)	1.43(B_c^+)
$\langle r_\infty^2 \rangle^{\frac{1}{2}}[11]$	-0.6025	0.427	0.427	1.836(B^+)	-1.29	-1.29(B_s^0)	1.84(B_c^+)

4 Discussion and Conclusion

We have analyzed elastic form factors and charge radii in a QCD inspired potential model using VIPT under two scenarios-Coulombic potential and linear potential as parent .

We summarize our achievements below:

- I. The form factor $eF(Q^2)$ decreases with the increase of Q^2 (as it should) for both the scenarios.
- II. The form factor is either very small (for D -sector mesons) or small (for B -sector mesons) with Coulombic parent as compared to those with linear parent.

The charge radii is also observed to be smaller with Coulombic parent as compared to linear parent.

- III. We use a fixed set of values for α_s under V -scheme[23-25] in the calculation , for example it is 0.693 for the D, D_s mesons which is larger than the value 0.261 for the B, B_s, B_c mesons. This consideration directly results in the unexpected smaller values of charge radii for D, D_s mesons as compared to the B, B_s, B_c mesons. Larger α_s values are responsible for smaller charge radius.

- IV. While checking the status of confinement as perturbation i.e. Coulombic parent (or Coulombic part as perturbation i.e. linear parent) we end up with a lower (or upper) limit on Q^2 . This allows us a useful range of Q^2 to show the variation of form factor which is shown in fig 1 and 2.

- V. In the present analysis, even with large b , the lower limit of Q^2 (for linear perturbation) are really small as shown in table 3 for fixed α_s . We have seen that for $\alpha_s = 0.693$, the lower limit of Q^2 for D, D_s mesons are respectively 0.0003, 0.0007, whereas with $\alpha_s = 0.261$, the lower limit of Q^2 for B, B_s, B_c mesons are respectively 0.05, 0.07, 0.205 . These values for B, B_s, B_c mesons will be lowered if we put $\alpha_s > 0.261$. This is clearly advantageous over the Dalgarno method with linear perturbation as done in Ref[1] where the formalism broke down for large b . Thus, this approach allows a large value of $\alpha_s(Q^2)$ in the limit $Q^2 \rightarrow 0$ even with large confinement , an important feature absent in Ref[1].

- VI. The Coulombic perturbation leads to an upper limit of Q^2 in this case (table 5) which allows us to use any value of $\alpha_s(Q^2)$ in the limit $Q^2 \rightarrow 0$.

- VII. Further, if we look at Eq.(1), consideration of different terms leads to different charge radii and the limiting values of Q^2 for both the cases. The charge radii and the lower limit of Q^2 decrease with more terms for the linear part as perturbation (i.e. Coulombic parent) whereas the charge radii increase and upper limit of Q^2 decreases for the Coulombic part as perturbation (i.e. linear parent).

- VIII. The infinite mass consideration in this work shows that the charge radii are larger than those for finite mass consideration to agree well with other models (table 6).

The above list as a whole suggests success of VIPT over the Dalgarno method [1, 22] as far as large confinement and limiting values of Q^2 are concerned. The difference in the values of form factors and charge radii for both the cases may be attributed to the use of same α_s (i.e. Q^2) under V -scheme for both the scenarios as the Coulombic potential is dominant for large Q^2 (i.e. low r) and the linear potential in the low Q^2 (i.e. large r) regime. We must note that we have used the low Q^2 assumption in the calculation of form factors and this clearly effects the upper limit of Q^2 corresponding to the validity of Coulombic perturbation (i.e. linear parent). The larger value of α_s for D -sector as compared to B -sector is also another point to take account of this. Although, the linear parent has shown more flexibility and hence is the better option than Coulombic parent in VIPT, but it has used terms up to a particular order in ' r ' in the integration involved with Airy function (which is an infinite series). This may lead to loss of certain information as far as physics is concerned. In the absence of any experimental results for these mesons, it is quite difficult to make a direct conclusion but there is clear indication that one must be careful in choosing the parameter $\alpha_s(Q^2)$ as well as the confinement parameter in the calculation of form factor and charge radius within the QCD framework.

The above discussion led to the conclusion that there is scope to use this approach in the study of meson decays. The lower and upper limit on Q^2 (i.e. range of Q^2) in this analysis may be useful in the experimental set up to investigate cross-section and form factor in future for these mesons. Further, from the model specific values of form factors and charge radii the method allows to investigate the behaviour of α_s w.r.t Q^2 in the nonperturbative regime of QCD.

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Expressions for q_i s :

$$q_1 = \frac{c_1'^2}{(4\mu^2\bar{\alpha}'^2 + Q_i^2)^{(1-\epsilon)}} \quad (27)$$

$$q_2 = A^2 \left[\frac{1}{(\mu^2\bar{\alpha}'^2 + Q_i^2)^{(1-\epsilon)}} - \frac{(3-2\epsilon)\mu\bar{\alpha}'}{(\mu^2\bar{\alpha}'^2 + Q_i^2)^{(1.5-\epsilon)}} + \frac{(4-2\epsilon)(3-2\epsilon)\mu^2\bar{\alpha}'^2}{(\mu^2\bar{\alpha}'^2 + Q_i^2)^{(2-\epsilon)}} \right] \quad (28)$$

$$\begin{aligned} q_3 = B^2 & \left[\frac{1}{\left(\frac{\mu^2\bar{\alpha}'^2}{9} + Q_i^2\right)^{(1-\epsilon)}} - \frac{4(3-2\epsilon)\mu\bar{\alpha}'}{3\left(\frac{\mu^2\bar{\alpha}'^2}{9} + Q_i^2\right)^{(1.5-\epsilon)}} + \right. \\ & \frac{16(4-2\epsilon)(3-2\epsilon)\mu^2\bar{\alpha}'^2}{27\left(\frac{\mu^2\bar{\alpha}'^2}{9} + Q_i^2\right)^{(2-\epsilon)}} - \frac{8(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\mu^3\bar{\alpha}'^3}{81\left(\frac{\mu^2\bar{\alpha}'^2}{9} + Q_i^2\right)^{(2.5-\epsilon)}} + \\ & \left. \frac{4(6-2\epsilon)(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\mu^4\bar{\alpha}'^4}{729\left(\frac{\mu^2\bar{\alpha}'^2}{9} + Q_i^2\right)^{(3-\epsilon)}} \right] \quad (29) \end{aligned}$$

$$\begin{aligned}
q_4 = D'^2 & \left[\frac{1}{16(\frac{\mu^2 \bar{\alpha}'^2}{4} + Q_i^2)^{(1-\epsilon)}} - \frac{3(3-2\epsilon)\mu \bar{\alpha}'}{32(\frac{\mu^2 \bar{\alpha}'^2}{4} + Q_i^2)^{(1.5-\epsilon)}} + \right. \\
& \frac{17(4-2\epsilon)(3-2\epsilon)\mu^2 \bar{\alpha}'^2}{256(\frac{\mu^2 \bar{\alpha}'^2}{4} + Q_i^2)^{(2-\epsilon)}} - \frac{19(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\mu^3 \bar{\alpha}'^3}{1536(\frac{\mu^2 \bar{\alpha}'^2}{4} + Q_i^2)^{(2.5-\epsilon)}} + \\
& \frac{7(6-2\epsilon)(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\mu^4 \bar{\alpha}'^4}{6144(\frac{\mu^2 \bar{\alpha}'^2}{9} + Q_i^2)^{(3-\epsilon)}} - \\
& \left. \frac{(7-2\epsilon)(6-2\epsilon)(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\mu^5 \bar{\alpha}'^5}{12288(\frac{\mu^2 \bar{\alpha}'^2}{4} + Q_i^2)^{(3.5-\epsilon)}} \right] \quad (30)
\end{aligned}$$

$$q_5 = 2c'_1 A \left[\frac{1}{(\frac{9\mu^2 \bar{\alpha}'^2}{4} + Q_i^2)^{(1-\epsilon)}} - \frac{(3-2\epsilon)\mu \bar{\alpha}'}{2(\frac{9\mu^2 \bar{\alpha}'^2}{4} + Q_i^2)^{(1.5-\epsilon)}} \right] \quad (31)$$

$$q_6 = 2c'_1 B \left[\frac{1}{(\frac{16\mu^2 \bar{\alpha}'^2}{9} + Q_i^2)^{(1-\epsilon)}} - \frac{2(3-2\epsilon)\mu \bar{\alpha}'}{3(\frac{16\mu^2 \bar{\alpha}'^2}{9} + Q_i^2)^{(1.5-\epsilon)}} + \frac{2(4-2\epsilon)(3-2\epsilon)\mu^2 \bar{\alpha}'^2}{27(\frac{16\mu^2 \bar{\alpha}'^2}{9} + Q_i^2)^{(2-\epsilon)}} \right] \quad (32)$$

$$\begin{aligned}
q_7 = 2c'_1 D' & \left[\frac{1}{4(\frac{25\mu^2 \bar{\alpha}'^2}{16} + Q_i^2)^{(1-\epsilon)}} - \frac{3(3-2\epsilon)\mu \bar{\alpha}'}{16(\frac{25\mu^2 \bar{\alpha}'^2}{16} + Q_i^2)^{(1.5-\epsilon)}} + \right. \\
& \frac{(4-2\epsilon)(3-2\epsilon)\mu^2 \bar{\alpha}'^2}{32(\frac{25\mu^2 \bar{\alpha}'^2}{16} + Q_i^2)^{(2-\epsilon)}} - \frac{(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\mu^3 \bar{\alpha}'^3}{768(\frac{25\mu^2 \bar{\alpha}'^2}{16} + Q_i^2)^{(2.5-\epsilon)}} \left. \right] \quad (33)
\end{aligned}$$

$$\begin{aligned}
q_8 = -2AB & \left[\frac{1}{(\frac{25\mu^2 \bar{\alpha}'^2}{36} + Q_i^2)^{(1-\epsilon)}} - \frac{5(3-2\epsilon)\mu \bar{\alpha}'}{6(\frac{25\mu^2 \bar{\alpha}'^2}{36} + Q_i^2)^{(1.5-\epsilon)}} + \right. \\
& \frac{20(4-2\epsilon)(3-2\epsilon)\mu^2 \bar{\alpha}'^2}{(27\frac{25\mu^2 \bar{\alpha}'^2}{36} + Q_i^2)^{(2-\epsilon)}} - \frac{(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\mu^3 \bar{\alpha}'^3}{27(\frac{25\mu^2 \bar{\alpha}'^2}{36} + Q_i^2)^{(2.5-\epsilon)}} \left. \right] \quad (34)
\end{aligned}$$

$$\begin{aligned}
q_9 = -2AD' & \left[\frac{1}{4(\frac{9\mu^2 \bar{\alpha}'^2}{16} + Q_i^2)^{(1-\epsilon)}} - \frac{5(3-2\epsilon)\mu \bar{\alpha}'}{16(\frac{9\mu^2 \bar{\alpha}'^2}{16} + Q_i^2)^{(1.5-\epsilon)}} + \right. \\
& \frac{(4-2\epsilon)(3-2\epsilon)\mu^2 \bar{\alpha}'^2}{8(\frac{9\mu^2 \bar{\alpha}'^2}{16} + Q_i^2)^{(2-\epsilon)}} - \frac{13(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\mu^3 \bar{\alpha}'^3}{768(\frac{9\mu^2 \bar{\alpha}'^2}{16} + Q_i^2)^{(2.5-\epsilon)}} + \\
& \left. \frac{(6-2\epsilon)(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\mu^4 \bar{\alpha}'^4}{1536(\frac{9\mu^2 \bar{\alpha}'^2}{16} + Q_i^2)^{(3-\epsilon)}} \right] \quad (35)
\end{aligned}$$

$$\begin{aligned}
q_{10} = 2BD' & \left[\frac{1}{4(\frac{49\mu^2 \bar{\alpha}'^2}{144} + Q_i^2)^{(1-\epsilon)}} - \frac{25(3-2\epsilon)\mu \bar{\alpha}'}{48(\frac{49\mu^2 \bar{\alpha}'^2}{144} + Q_i^2)^{(1.5-\epsilon)}} + \right. \\
& \frac{151(4-2\epsilon)(3-2\epsilon)\mu^2 \bar{\alpha}'^2}{864(\frac{25\mu^2 \bar{\alpha}'^2}{144} + Q_i^2)^{(2-\epsilon)}} - \frac{27.66(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\mu^3 \bar{\alpha}'^3}{768(\frac{25\mu^2 \bar{\alpha}'^2}{144} + Q_i^2)^{(2.5-\epsilon)}} + \\
& \frac{3.66(6-2\epsilon)(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\mu^4 \bar{\alpha}'^4}{1152(25\frac{\mu^2 \bar{\alpha}'^2}{144} + Q_i^2)^{(3-\epsilon)}} - \\
& \left. \frac{(7-2\epsilon)(6-2\epsilon)(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\mu^4 \bar{\alpha}'^4}{10368(25\frac{\mu^2 \bar{\alpha}'^2}{144} + Q_i^2)^{(3.5-\epsilon)}} \right] \quad (36)
\end{aligned}$$

Expressions for r_i s:

$$r_1 = 3c_1^2(1 + \frac{m_i}{m_j})^{-2}(4\mu^2\bar{\alpha}^2)^{\epsilon-2}(2-2\epsilon) \quad (37)$$

$$r_2 = 3A^2(1 + \frac{m_i}{m_j})^{-2}[(2-2\epsilon)(4\mu^2\bar{\alpha}^2)^{\epsilon-2} - 3\mu\bar{\alpha}'(3-2\epsilon)^2(\mu^2\bar{\alpha}^2)^{\epsilon-2.5} + 0.75\mu^2\bar{\alpha}^2(4-2\epsilon)^2(3-2\epsilon)(\mu^2\bar{\alpha}^2)^{\epsilon-3}] \quad (38)$$

$$r_3 = 3B^2(1 + \frac{m_i}{m_j})^{-2}[(\frac{4\mu^2\bar{\alpha}'^2}{9})^{\epsilon-2}(2-2\epsilon) - 4\mu\bar{\alpha}'(3-2\epsilon)^2(\frac{4\mu^2\bar{\alpha}'^2}{9})^{\epsilon-2.5} + \frac{16\mu^2\bar{\alpha}'^2}{9}(4-2\epsilon)^2(3-2\epsilon)(\frac{4\mu^2\bar{\alpha}'^2}{9})^{\epsilon-3} - \frac{8\mu^3\bar{\alpha}'^3}{81}(5-2\epsilon)^2(4-2\epsilon)(3-2\epsilon)(\frac{4\mu^2\bar{\alpha}'^2}{9})^{\epsilon-3.5} + \frac{4\mu^4\bar{\alpha}'^4}{729}(6-2\epsilon)^2(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)(\frac{4\mu^2\bar{\alpha}'^2}{9})^{\epsilon-4}] \quad (39)$$

$$r_4 = 3D'^2(1 + \frac{m_i}{m_j})^{-2}[(\frac{\mu^2\bar{\alpha}'^2}{4})^{\epsilon-2}(2-2\epsilon) - \frac{9\mu\bar{\alpha}'}{32}(3-2\epsilon)^2(\frac{\mu^2\bar{\alpha}'^2}{4})^{\epsilon-2.5} + \frac{57\mu^2\bar{\alpha}'^2}{32}(4-2\epsilon)^2(3-2\epsilon)(\frac{\mu^2\bar{\alpha}'^2}{4})^{\epsilon-3} - \frac{19\mu^3\bar{\alpha}'^3}{512}(5-2\epsilon)^2(4-2\epsilon)(3-2\epsilon)(\frac{\mu^2\bar{\alpha}'^2}{4})^{\epsilon-3.5} + \frac{21\mu^4\bar{\alpha}'^4}{6144}(6-2\epsilon)^2(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)(\frac{\mu^2\bar{\alpha}'^2}{4})^{\epsilon-4} - \frac{3\mu^5\bar{\alpha}'^5}{12288}(7-2\epsilon)^2(6-2\epsilon)(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)(\frac{4\mu^2\bar{\alpha}'^2}{9})^{\epsilon-4.5}] \quad (40)$$

$$r_5 = 2c_1'A(1 + \frac{m_i}{m_j})^{-2}[3(\frac{9\mu^2\bar{\alpha}^2}{4})^{\epsilon-2}(2-2\epsilon) - 1.5\mu\bar{\alpha}'(3-2\epsilon)^2(\frac{9\mu^2\bar{\alpha}^2}{4})^{\epsilon-2.5}] \quad (41)$$

$$r_6 = 2c_1'B(1 + \frac{m_i}{m_j})^{-2}[3(\frac{16\mu^2\bar{\alpha}'^2}{9})^{\epsilon-2}(2-2\epsilon) - 2\mu\bar{\alpha}'(3-2\epsilon)^2(\frac{16\mu^2\bar{\alpha}'^2}{9})^{\epsilon-2.5} + \frac{2\mu^2\bar{\alpha}'^2}{9}(4-2\epsilon)^2(3-2\epsilon)(\frac{16\mu^2\bar{\alpha}'^2}{9})^{\epsilon-3}] \quad (42)$$

$$r_7 = 2c_1'D'(1 + \frac{m_i}{m_j})^{-2}[(\frac{25\mu^2\bar{\alpha}'^2}{16})^{\epsilon-2}\frac{3(2-2\epsilon)}{4} - \frac{9\mu\bar{\alpha}'(3-2\epsilon)^2}{16}(\frac{25\mu^2\bar{\alpha}'^2}{16})^{\epsilon-2.5} + \frac{3\mu^2\bar{\alpha}'^2}{32}(4-2\epsilon)^2(3-2\epsilon)(\frac{25\mu^2\bar{\alpha}'^2}{16})^{\epsilon-3} - \frac{3\mu^3\bar{\alpha}'^3}{768}(5-2\epsilon)^2(4-2\epsilon)(3-2\epsilon)(\frac{25\mu^2\bar{\alpha}'^2}{16})^{\epsilon-3.5}] \quad (43)$$

$$\begin{aligned}
r_8 = & -2AB(1 + \frac{m_i}{m_j})^{-2}[(\frac{25\mu^2\bar{\alpha}'^2}{36})^{\epsilon-2}3(2-2\epsilon) - \\
& 2.5\mu\bar{\alpha}'(3-2\epsilon)^2(\frac{25\mu^2\bar{\alpha}'^2}{36})^{\epsilon-2.5} + \\
& \frac{20\mu^2\bar{\alpha}'^2}{9}(4-2\epsilon)^2(3-2\epsilon)(\frac{25\mu^2\bar{\alpha}'^2}{36})^{\epsilon-3} - \\
& \frac{\mu^3\bar{\alpha}'^3}{9}(5-2\epsilon)^2(4-2\epsilon)(3-2\epsilon)(\frac{25\mu^2\bar{\alpha}'^2}{36})^{\epsilon-3.5}] \quad (44)
\end{aligned}$$

$$\begin{aligned}
r_9 = & -2AD'(1 + \frac{m_i}{m_j})^{-2}[0.75(\frac{9\mu^2\bar{\alpha}'^2}{16})^{\epsilon-2}(2-2\epsilon) - \\
& \frac{15\mu\bar{\alpha}'}{16}(3-2\epsilon)^2(\frac{9\mu^2\bar{\alpha}'^2}{16})^{\epsilon-2.5} + \\
& \frac{3\mu^2\bar{\alpha}'^2}{8}(4-2\epsilon)^2(3-2\epsilon)(\frac{9\mu^2\bar{\alpha}'^2}{16})^{\epsilon-3} - \\
& \frac{13\mu^3\bar{\alpha}'^3}{256}(5-2\epsilon)^2(4-2\epsilon)(3-2\epsilon)(\frac{9\mu^2\bar{\alpha}'^2}{16})^{\epsilon-3.5} + \\
& \frac{\mu^4\bar{\alpha}'^4}{512}(6-2\epsilon)^2(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)(\frac{9\mu^2\bar{\alpha}'^2}{16})^{\epsilon-4}] \quad (45)
\end{aligned}$$

$$\begin{aligned}
r_{10} = & 2BD'(1 + \frac{m_i}{m_j})^{-2}[0.75(\frac{49\mu^2\bar{\alpha}'^2}{144})^{\epsilon-2}(2-2\epsilon) - \\
& \frac{17\mu\bar{\alpha}'}{16}(3-2\epsilon)^2(\frac{49\mu^2\bar{\alpha}'^2}{144})^{\epsilon-2.5} + \\
& \frac{151\mu^2\bar{\alpha}'^2}{288}(4-2\epsilon)^2(3-2\epsilon)(\frac{49\mu^2\bar{\alpha}'^2}{144})^{\epsilon-3} - \\
& \frac{83\mu^3\bar{\alpha}'^3}{768}(5-2\epsilon)^2(4-2\epsilon)(3-2\epsilon)(\frac{49\mu^2\bar{\alpha}'^2}{144})^{\epsilon-3.5} + \\
& \frac{33\mu^4\bar{\alpha}'^4}{3456}(6-2\epsilon)^2(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)(\frac{49\mu^2\bar{\alpha}'^2}{144})^{\epsilon-4} - \\
& \frac{\mu^5\bar{\alpha}'^5}{3456}(7-2\epsilon)^2(6-2\epsilon)(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)(\frac{49\mu^2\bar{\alpha}'^2}{144})^{\epsilon-4.5}] \quad (46)
\end{aligned}$$

Expressions for p_i s:

$$p_1 = \frac{c'_1}{(\frac{\mu^2\bar{\alpha}'^2}{4} + Q_i^2)^{\frac{(3-\epsilon)}{2}}} \quad (47)$$

$$p_2 = A[\frac{1}{(\frac{\mu^2\bar{\alpha}'^2}{4} + Q_i^2)^{\frac{(3-\epsilon)}{2}}} - \frac{(3-2\epsilon)\mu\bar{\alpha}'}{2(\frac{\mu^2\bar{\alpha}'^2}{4} + Q_i^2)^{\frac{(4-\epsilon)}{2}}}] \quad (48)$$

$$\begin{aligned}
p_3 = & B[\frac{1}{(\frac{\mu^2\bar{\alpha}'^2}{9} + Q_i^2)^{\frac{(3-\epsilon)}{2}}} - \frac{0.67(3-2\epsilon)\mu\bar{\alpha}'}{(\frac{\mu^2\bar{\alpha}'^2}{9} + Q_i^2)^{\frac{(4-\epsilon)}{2}}} - \\
& \frac{2(4-2\epsilon)(3-2\epsilon)\mu^2\bar{\alpha}'^2}{27(\frac{\mu^2\bar{\alpha}'^2}{9} + Q_i^2)^{\frac{(5-\epsilon)}{2}}}] \quad (49)
\end{aligned}$$

$$p_4 = D' \left[\frac{0.25}{\left(\frac{\mu^2 \bar{\alpha}'^2}{16} + Q_i^2\right)^{\frac{(3-\epsilon)}{2}}} - \frac{3(3-2\epsilon)\mu \bar{\alpha}'}{16\left(\frac{\mu^2 \bar{\alpha}'^2}{16} + Q_i^2\right)^{\frac{(4-\epsilon)}{2}}} - \frac{(4-2\epsilon)(3-2\epsilon)\mu^2 \bar{\alpha}'^2}{32\left(\frac{\mu^2 \bar{\alpha}'^2}{16} + Q_i^2\right)^{\frac{(5-\epsilon)}{2}}} - \frac{(5-2\epsilon)(4-2\epsilon)(3-2\epsilon)\mu^3 \bar{\alpha}'^3}{768\left(\frac{\mu^2 \bar{\alpha}'^2}{16} + Q_i^2\right)^{\frac{(6-\epsilon)}{2}}} \right] \quad (50)$$

Expressions for p'_i s :

$$p'_1 = n_1 \times (\bar{b}' \mu)^{\frac{2}{3}} \quad (51)$$

$$p'_2 = n_2 \times (\bar{b}' \mu)^{\frac{2}{3}} \quad (52)$$

$$p'_3 = n_3 \times (\bar{b}' \mu)^{\frac{2}{3}} \quad (53)$$

$$p'_4 = n_4 \times (\bar{b}' \mu)^{\frac{2}{3}} \quad (54)$$

$$p'_5 = n_5 \times (\bar{b}' \mu)^{\frac{2}{3}} \quad (55)$$

$$p'_6 = n_6 \times (\bar{b}' \mu)^{\frac{2}{3}} \quad (56)$$

$$p'_7 = n_7 \times (\bar{b}' \mu)^{\frac{2}{3}} \quad (57)$$

$$p'_8 = n_8 \times (\bar{b}' \mu)^{\frac{2}{3}} \quad (58)$$

Each of the constants n_1, n_2, \dots, n_8 are different for different mesons and they have been obtained by numerical integration.